**Algebra 2 Unit 2: Exponential Functions Activity**

***m & m’s*** *and Modeling Exponential Functions Activity*

# Description---

Given a number of m & m’s chocolate candies and the outlined procedure, approximations for exponential growth and exponential decay can be modeled. The method for exponential growth is performed with the students while the procedure for exponential decay is left as an exercise.

Of course, m & m’s candies are not the only pedagogical instruments that could be used. Coins that could land heads or tails or thumbtacks that could land point up or point down could be used as well. Any object with a binomial characteristic would do. A simulation could also be performed using a graphing calculator if the students and instructor are well versed. m & m’s were chosen because they are easily handled, readily available, not cost prohibitive and absolutely delicious! **Please devour after the project is concluded.**

# Exponential Functions---

An exponential function with base b has the form  where a is a positive real number and is called the *initial amount.* The base b, which is called the *growth factor,* represents the factor by which the initial amount grows. The growth factor b must be greater than 0 and cannot equal 1. If *b > 1*, then a situation called *exponential growth* occurs. If *0<b<1*, then  is a decreasing function and *exponential decay* occurs. Graph both an exponential growth and exponential decay graph below.

# Procedure for modeling exponential growth with m & m’s---

1. Start with a given number of m & m’s, say three. Record this amount as the population under

trial 0 in the table provided.

1. Place the m & m’s in cup, gently shake them, then dump them onto your napkin or a piece of paper.
2. Count the number of m & m’s that are “m up”. For each one that is “m up”, add twice that number back into the cup along with those that were dumped. Why twice? That is the growth factor we are trying to model. Record the new total in your table. For example, if you had three m & m’s in your cup and two were “m up” after you dumped them, you would add four more to the cup, for a total of seven.
3. Replace the m & m’s in the cup and repeat steps 2 and 3 until you run out of m & m’s to add, making sure to record your data in the table throughout the process.

Let’s try exponential growth together!

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Trial | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| Population |  |  |  |  |  |  |  |  |

Let’s graph our results.

# Determining the exponential equation---

1. First of all, we should verify that the function is exponential. Begin by finding the ratio between the populations of two consecutive trials. You then want to check the ratios of other consecutive trials and verify that they are reasonably close. Pick an arbitrary student’s data and check for constant ratios.
2. Once you have checked to see if the function is exponential, you will want to derive the exponential equation. One method will employ a system of equations in which you will solve for a and b. Pick two ordered pairs from a student’s data. Let’s say a student has points (1,7) and (4,45).

At this point you will set up a system of equations. Using  and substituting x and y values from the two ordered pairs gives you  and . Dividing the first equation by the second yields . Taking the cube root of both sides gives b1.86 which is reasonably close to 2 and that is the growth factor we were trying to get with the

m & m’s. We can then substitute the value of b back into either equation and solve for a. For our example, a3.76 and yes, it was supposed to equal 3 but there will be some small error along the way.

So for our student’s data, his exponential equation was **y = (3.76)(1.86x)**. Theoretically, it should have been **y = (3)(2x)** .

1. A graphing calculator can also give the exponential equation by using regression techniques. The students will place the number of trials in one list and the populations into another list. After looking at scatter plot of the data to see if it looks exponential, have the calculator find the exponential regression equation. For a TI-83+, to enter data into lists:

Press **STAT**, **EDIT** and enter your data. Use L1 for trials and L2 for population.

To look at the scatter plot Press **2nd**, **Y=,** and select a plot number ( 1,2,or 3). Turn the plot on and choose the first of the six types. Your lists should match your data, x-values should be L1 and y-values should be L2. Choose a mark and press **ZOOM 9** (Zoom Statistics) to view the graph.

To get the exponential equation, press **STAT** and arrow over to **CALC** then locate exponential regression (ExpReg). Press **ENTER** and then press **2nd** **1** followed by a comma and **2nd** **2** another comma and then **VARS**. Go over to **Y-VARS** and press **ENTER** twice. Your screen should appear like this: **ExpReg L1, L2 , Y1** . This will automatically place the line of your equation with the scatter plot. Pressing **ENTER** will give you your exponential function’s equation. You may hit **ZOOM 9** to view how well it fits your data.

1. Try your data and see if it is exponential and then determine the equation. What is the theoretical equation? How does your equation compare?
2. Calculate the average  and average  for the class. How do these compare with #4?

Exponential Decay

1. If you were trying to model exponential decay with the m & m’s, how would you go about it?

Describe in detail.

1. This is your procedure. Try it.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Trial | 0 | 1 | 2 | 3 | 4 | 5 |  |  |
| Population |  |  |  |  |  |  |  |  |

1. Derive your exponential decay equation with your calculator.
2. What is the theoretical equation? How does your equation compare?
3. Calculate the average  and average  for the class. How do these compare with #4?
4. Hypothetically, if you had two data points of (2, 13) and (5, 91), answer the following:

i) What type of exponential function would you have? Growth or decay? Why?

ii) Derive the equation based on the two given points.

iii) Predict the population on the 4th trial based on the equation from (ii).

1. Given these data points: (0,150) , (1,70) , (2,28) , (3,18) , (4,7) , (5,2) , ( 6,1) , derive the exponential model by hand.
2. If you would add triple the number of m & m’s that were “m up” back into the cup along with those previously dumped, what exponential equation would you be trying to model? Explain.